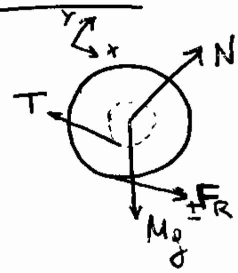
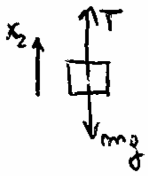
PROBLEMA 1

a)





NEWTON:

$$\boxed{Mg \operatorname{sen}\alpha \pm F_R - T = M a_{cm}} \ (1) \qquad \boxed{T - mg = m a_2} \ (2)$$

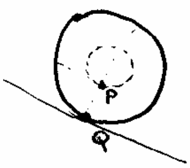
$$N - Mg\cos\alpha = 0$$

MOMENTOS:

$$I_{cm}\dot{\vec{\Omega}} = \frac{MR^2}{2}\dot{\Omega}\,\hat{z} = -\frac{R}{2}\hat{y} \wedge (-T)\hat{x} + (-R)\hat{y} \wedge (\pm F_R)\hat{x}$$

$$\boxed{\frac{MR}{2}\dot{\Omega} = -\frac{T}{2} \pm F_R} \ (3)$$

VINCULOS:



$v_Q = 0$ x RODADURA $\rightarrow$ $\vec{v}_{cm} = \cancel{\vec{v}_Q}^{=0} + \vec{\Omega} \times R\hat{y}$

$v_{cm}\hat{x} = \Omega\hat{z} \times R\hat{y}$

$v_{cm} = -\Omega R$

$$\boxed{a_{cm} = -\dot{\Omega}R} \ (4)$$

$v_P = v_2$ x LA SOGA

$\downarrow$

$$\vec{v}_{cm} = \vec{v}_P + \vec{\Omega} \times \frac{R}{2}\hat{y}$$

$$v_{cm}\hat{x} = v_2\hat{x} + \Omega\hat{z} \times \frac{R}{2}\hat{y} \longrightarrow v_{cm} = v_2 - \frac{\Omega R}{2}$$

$$\boxed{a_{cm} = a_2 - \frac{\dot{\Omega}R}{2}} \ (5)$$

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b) $(4) \rightarrow (5) \rightarrow \quad a_2 = \frac{a_{cm}}{2} \quad \Rightarrow \quad (2) \rightarrow T = \frac{m a_{cm}}{2} + mg$

$(3) \rightarrow \pm F_R = \frac{MR\dot{\Omega}}{2} + \frac{T}{2}$

$\pm F_R = -\frac{M a_{cm}}{2} + \frac{T}{2}$

$(1) \longrightarrow Mg\operatorname{sen}\alpha - \frac{M a_{cm}}{2} + \frac{T}{2} - T = M a_{cm} \rightarrow Mg\operatorname{sen}\alpha - \frac{T}{2} = \frac{3}{2}M a_{cm}$

$\longrightarrow Mg\operatorname{sen}\alpha - \frac{m}{4}a_{cm} + \frac{mg}{2} = \frac{3}{2}M a_{cm}$

$$Mg\operatorname{sen}\alpha + \frac{mg}{2} = \left(\frac{3}{2}M + \frac{m}{4}\right)a_{cm} \quad \Rightarrow \quad \boxed{a_{cm} = 2g\left(\frac{2M\operatorname{sen}\alpha - m}{6M + m}\right)}$$

PROBLEMA 1 (CONT)

$$\boxed{\vec{a}_{CM} = 2g\left(\frac{2M\,\text{sen}\,\alpha - m}{6M+m}\right)\hat{x}}$$

$$a_{CM} = -\dot{\Omega} R \quad \Rightarrow \quad \boxed{\vec{\dot{\Omega}} = -\frac{2g}{R}\left(\frac{2M\,\text{sen}\,\alpha - m}{6M+m}\right)\hat{z}}$$

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c) $M = 2m \quad \alpha = 30^\circ$

PARA QUE BAJE POR EL PLANO: $2M\,\text{sen}\,\alpha - m > 0$

$$4m \cdot \frac{1}{2} - m = m > 0 \quad \checkmark$$

$(1) \rightarrow Mg\,\text{sen}\,\alpha + F_{RE} - T = M a_{CM} \qquad (2) \rightarrow T = m\frac{a_{CM}}{2} + mg$

$$Mg\,\text{sen}\,\alpha + F_{RE} - \frac{m}{2}a_{CM} - mg = M a_{CM}$$

$$a_{CM} = 2g\left(\frac{m}{13m}\right) = \frac{2}{13}g$$

$$2mg\,\frac{1}{2} + F_{RE} - \frac{1}{13}mg - mg = \frac{4}{13}mg$$

$$F_{RE} = \frac{5}{13}mg \leq \mu_E N = \mu_E\, 2mg\,\frac{\sqrt{3}}{2} \rightarrow \boxed{\mu_E \geq \frac{5}{13\sqrt{3}}}$$

PROBLEMA 2 :

a) ANTES : x NEWTON: $+F_g = +m r\dot{\theta}^2$ $\qquad \vec{v}_s = r\dot{\theta}\,\hat{\theta} = 3R\dot{\theta}\,\hat{\theta}$

$$\frac{GMm}{(3R)^2} = m\,3R\dot{\theta}^2 = m\frac{v_s^2}{3R} \Rightarrow \boxed{\vec{v}_s = \sqrt{\frac{GM}{3R}}\,\hat{\theta}}$$

DESPUES : AL SER INDEPENDIENTE DE m,
EL RESULTADO ANTERIOR VALE PARA m_A $\Rightarrow \boxed{\vec{v}_A = -\sqrt{\frac{GM}{3R}}\,\hat{\theta}}$

POR CONSERVACION DE $\vec{L}$ (Y/o DE $\vec{P}$):

$$\hat{\theta})\quad m\sqrt{\frac{GM}{3R}} = -m_A\sqrt{\frac{GM}{3R}} + m_B v_B$$

$$\rightarrow \frac{(m+m_A)}{m_B}\sqrt{\frac{GM}{3R}} = v_B \xrightarrow{m=m_A+m_B} \boxed{\vec{v}_B = \left(1+\frac{2m_A}{m_B}\right)\sqrt{\frac{GM}{3R}}\,\hat{\theta}}$$

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b) PARA m_B, LUEGO DE LA EXPLOSION:

$$E_B = -\frac{GMm_B}{3R} + \frac{1}{2}m_B\left(1+\frac{2m_A}{m_B}\right)^2\frac{GM}{3R} = \frac{GMm_B}{3R}\left[\frac{1}{2}\left(1+\frac{2m_A}{m_B}\right)^2-1\right]$$

PARA "ESCAPAR" $E_B > 0 \Rightarrow \frac{1}{2}\left(1+\frac{2m_A}{m_B}\right)^2 > 1 \Rightarrow \boxed{\frac{m_A}{m_B} > \frac{\sqrt{2}-1}{2}}$

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c) $m_A = m_B = \frac{m}{2} \Rightarrow E_B = \frac{7}{6}\frac{GMm_B}{R}$

$$E_B = -\frac{GMm_B}{r_0} + \frac{1}{2}m_B r_0^2\dot{\theta}_0^2$$ $\rightarrow$ SIENDO r_0 LA DIST. DE MAX. ACERCAMIENTO ($\dot{r}=0$)

$$\frac{7}{6}\frac{GM}{R} = -\frac{GM}{r_0} + \frac{1}{2}(3R)^2\frac{v_B^2}{r_0^2} = -\frac{GM}{r_0} + \frac{27}{2}R\frac{GM}{r_0^2}$$

$$\frac{7}{6}\frac{1}{R} = -\frac{1}{r_0} + \frac{27}{2}\frac{R}{r_0^2} \Rightarrow \frac{7}{6}r_0^2 + Rr_0 - \frac{27}{2}R^2 = 0$$

$$\Rightarrow r_0 = \frac{-R \pm\sqrt{R^2+4\frac{7}{6}\frac{27}{2}R^2}}{7/3} = \begin{cases} 3R \longrightarrow \boxed{r_0 = 3R} \\ -\frac{27}{7}R \rightarrow \text{NO TIENE SENTIDO FISICO}\end{cases}$$

$\vec{L}_B = 3R\hat{r}\times m_B v_B\hat{\theta} = 3R m_B v_B\,\hat{z}$

$\vec{L}_B = r_0\hat{r}\times m_B r_0\dot{\theta}_0\hat{\theta} = m_B r_0^2\dot{\theta}_0\,\hat{z}$

$\Rightarrow \dot{\theta}_0 = \frac{3Rv_B}{r_0^2}$

$v_B = \sqrt{\frac{3GM}{R}}$

ESTE RESULTADO SE PODRIA HABER ANTICIPADO VIENDO QUE $m_A = m_B = \frac{m}{2}$ SATISFACE LA RELACION HALLADA EN (b), Y m_B ESCAPA DESDE $r=3R$ CON $\dot{r}=0$

PROBLEMA 3:

a)
$$\frac{d\vec{L}_1^{(o)}}{dt} = r_1 \hat{r}_1 \times (-T-F_e)\hat{r}_1 = 0 \Rightarrow \boxed{\vec{L}_1^{(o)} = CTE}$$
$$\frac{d\vec{L}_2^{(o)}}{dt} = r_2 \hat{r}_2 \times (-T)\hat{r}_2 = 0 \Rightarrow \boxed{\vec{L}_2^{(o)} = CTE}$$
$$\Rightarrow \boxed{\vec{L}^{(o)} = CTE}$$

$$\frac{d\vec{P}}{dt} = \underbrace{\vec{T}_1 + \vec{F}_e}_{= \frac{d\vec{P}_1}{dt} \neq 0} + \underbrace{\vec{T}_2}_{= \frac{d\vec{P}_2}{dt} \neq 0} \neq 0 \Rightarrow \boxed{\vec{P} \neq CTE}$$

$$\Rightarrow \boxed{\vec{P}_1, \vec{P}_2 \neq CTE}$$

$|\vec{T}_1| = |\vec{T}_2|$ x SOGA IDEAL

PERO TIENEN $\neq$ DIRECCION

$$\Delta E_1 = W_{T_1} \neq 0 \Rightarrow \boxed{E_1 \neq CTE}$$
$$\Delta E_2 = W_{T_2} \neq 0 \Rightarrow \boxed{E_2 \neq CTE}$$

$$d = r_1 + r_2$$
$$0 = \dot{r}_1 + \dot{r}_2 \qquad dr_1 = -dr_2$$

$$\Delta E = W_{T_1} + W_{T_2} = \int \vec{T}_1 d\vec{r}_1 + \int \vec{T}_2 d\vec{r}_2 = \int T\, dr_1 + \int T\, dr_2 = 0 \Rightarrow \boxed{E = CTE}$$

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b) $\ell = 2\ell_0$:
$$\vec{L}_1 = 2\ell_0 \hat{r}_1 \times m_1 v_0 \hat{\theta}_1 = 2 m_1 \ell_0 v_0 \hat{z}$$
$$\vec{L}_2 = (d-2\ell_0)\hat{r}_2 \times 0 = 0$$

$$E = \frac{1}{2} K \ell_0^2 + \frac{1}{2} m_1 v_0^2$$

$\ell = \ell_0$:
$$\vec{L}_1 = \ell_0 \hat{r}_1 \times m_1 v_1^\theta \hat{\theta}_1 = m_1 \ell_0 v_1^\theta \hat{z} = 2 m_1 \ell_0 v_0 \hat{z} \Rightarrow \boxed{v_1^\theta = 2v_0}$$
$$\vec{L}_2 = (d-\ell_0)\hat{r}_2 \times m_2 v_2^\theta \hat{\theta}_2 = (d-\ell_0) m_2 v_2^\theta = 0 \Rightarrow \boxed{v_2^\theta = 0} \qquad \dot{\theta}_2 = 0 \ \forall t$$

$$E = \frac{1}{2} m_1 v_r^{1\,2} + \frac{1}{2} m_1 v_\theta^{1\,2} + \frac{1}{2} m_2 v_r^{2\,2} \qquad v_r^2 = -v_r^1$$

$$E = \frac{1}{2}(m_1+m_2) v_r^{1\,2} + \frac{1}{2} m_1 4 v_0^2 = \frac{1}{2} K \ell_0^2 + \frac{1}{2} m_1 v_0^2$$

$$\Rightarrow \boxed{v_r^1 = \pm \sqrt{\frac{K\ell_0^2 - 3 m_1 v_0^2}{m_1+m_2}}} \qquad \boxed{v_r^2 = \mp \sqrt{\frac{K\ell_0^2 - 3 m_1 v_0^2}{m_1+m_2}}}$$

PROBLEMA 3: (CONT)

c) $$E = \frac{1}{2}k(r_1 - l_0)^2 + \frac{1}{2}(m_1+m_2)\dot{r}_1^2 + \frac{1}{2}m_1 r_1^2 \dot{\theta}_1^2$$

$$\vec{L}_1 = r_1 \hat{r}_1 \times m_1(\dot{r}_1 \hat{r} + r_1\dot{\theta}_1 \hat{\theta}) = m_1 r_1^2 \dot{\theta}_1 \hat{z} = 2m_1 l_0 v_0 \hat{z}$$

$$\dot{\theta}_1 = \frac{2l_0 v_0}{r_1^2}$$

$$E = \frac{1}{2}(m_1+m_2)\dot{r}_1^2 + \underbrace{\frac{1}{2}k(r_1-l_0)^2 + 2m_1 \frac{l_0^2 v_0^2}{r_1^2}}_{\rightarrow V_{ef}}$$

MOV. CIRCULAR: $\left.\frac{dV_{ef}}{dr_1}\right|_{r_0} = 0$

$$\frac{dV_{ef}}{dr_1} = k(r_1 - l_0) - 4\frac{m_1 l_0^2 v_0^2}{r_1^3}$$

$$\left.\frac{dV_{ef}}{dr_1}\right|_{r_1=2l_0} = kl_0 - \frac{4m_1 l_0^2 v_0^2}{8l_0^3} = kl_0 - \frac{m_1 v_0^2}{2l_0} = 0$$

$$\Rightarrow \boxed{v_0^2 = \frac{2kl_0^2}{m_1}}$$

NEWTON:

$$\left.\begin{array}{lll} m_1 & \hat{r}_1) & -kl_0 - T = -m_1 2l_0 \dot{\theta}_1^2 \\ m_2 & \hat{r}_2) & -T = 0 \end{array}\right] \rightarrow kl_0 = m_1 2l_0 \dot{\theta}_1^2$$

$$\dot{\theta}_1 = \frac{v_0}{2l_0}$$

$$kl_0 = m_1 2l_0 \frac{v_0^2}{4l_0^2} \Longrightarrow \boxed{v_0^2 = \frac{2kl_0^2}{m_1}}$$