Magnetohydrodynamic flows of conducting liquids in divergent-convergent channels
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We show that the full set of magnetohydrodynamic equations for resistive and viscous, incompressible fluids allows, in cylindrical coordinates, an exact reduction to a pair of coupled, nonlinear, ordinary differential equations for two scalar potentials, the stream and the magnetic flux function. The ordinary differential equations admit solutions that represent self-similar magnetohydrodynamic flows in channels, bounded by non-parallel plane walls, intersecting on the z-axis, akin to the Jeffery-Hamel flows of non-conducting fluids. We consider the case in which an external current, flows along the z-axis, and exerts a body force that controls the flow. In this case only one nonlinear ordinary differential equation governs the solution. Besides the Reynolds number ($R_e$) and the angle of the walls, two other non dimensional parameters determine the solutions, the magnetic Reynolds number ($R_m$), and the Hartmann number ($H_a$). We give a preliminary study of the properties of a high resistivity regime, in which the Hartmann number becomes important. The main result is that for $H_a$ larger than 2 the flow reversal near the walls, which is a typical feature of channels with $H_a=0$ and $R_e\sim O(1)$ or larger, tends to disappear. Moderate values of $H_a\sim 6$ are sufficient to suppress flow reversal up to $R_e \sim 25$ for angular widths of the channel as large as 240 degrees. The configuration studied here may be of interest for applications in flow control of liquid metals, and in industrial treatment of molten metals.